

## Flavor symmetry for quarks and leptons

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**ABSTRACT:** Present data on neutrino masses and mixing favor the highly symmetric tribimaximal neutrino mixing matrix which suggests an underlying flavor symmetry. A systematic study of non-abelian finite groups of order  $g \leq 31$  reveals that tribimaximal mixing can be derived not only from the well known tetrahedral flavor symmetry  $T \equiv A_4$ , but also by using the binary tetrahedral symmetry  $T' \equiv SL_2(F_3)$  which does not contain the tetrahedral group as a subgroup.  $T'$  has the further advantage that it can also neatly accommodate the quark masses including a heavy top quark.

**KEYWORDS:** Global Symmetries, Discrete and Finite Symmetries.

In this letter we propose a flavor symmetry for quarks and leptons.

We shall consider only three left-handed neutrinos at first. The Majorana mass matrix  $\mathcal{M}$  is a  $3 \times 3$  unitary symmetric matrix and without CP violation has six real parameters. Let write the diagonal form as  $\mathbf{M} = \text{diag}(m_1, m_2, m_3)$ , related to the flavor basis  $\mathcal{M}$  by  $\mathbf{M} = U^T \mathcal{M} U$  where  $U$  is orthogonal. It is the form of  $\mathcal{M} = U \mathbf{M} U^T$  and  $U$  which are the targets of lepton flavor physics. One technique for analysis of  $\mathcal{M}$  is to assume texture zeros [1–3] in  $\mathcal{M}$  and this gives rise to relationships between the mass eigenvalues  $m_i$  and the mixing angles  $\theta_{ij}$ . For example, it was shown in [2] that  $\mathcal{M}$  cannot have as many as three texture zeros out of a possible six but can have two. A quite different interesting philosophy is that neutrino masses may arise from breaking of lorentz invariance [4]. Clearly, a wide range of approaches is being aimed at the problem.

In the present study we focus on a symmetric texture for  $\mathcal{M}$  with only four independent parameters, of the form

$$\mathcal{M} = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix} \quad (1)$$

The  $\mathcal{M}$  can be reached from a diagonal  $\mathbf{M}$  by the orthogonal transformation

$$U = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12}/\sqrt{2} & \cos\theta_{12}/\sqrt{2} & -1/\sqrt{2} \\ -\sin\theta_{12}/\sqrt{2} & \cos\theta_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \quad (2)$$

where one commits to a relationship between  $\theta_{12}$  and the four parameters in eq. (1), namely

$$\tan 2\theta_{12} = 2\sqrt{2B(A - C - D)^{-1}} \quad (3)$$

Written in the standard PMNS form [5]

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}e^{-i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (4)$$

this ansatz requires that  $\theta_{23} = \pi/4$  and  $\theta_{13} = 0$ , both of which are consistent with present data. These values of maximal  $\theta_{23}$  and vanishing  $\theta_{13}$  are presumably only approximate but departures therefrom, if they show up in future experiments, could be accommodated by higher order corrections.

To arrive at tribimaximal mixing [6–11], one more parameter  $\theta_{12}$  in eq. (2) is assigned such that the entries of the second column are equal, i.e.  $\sin\theta_{12} = \cos\theta_{12}/\sqrt{2}$  which implies that  $\tan^2\theta_{12} = 1/2$ . Experimentally  $\theta_{12}$  is non-zero and over  $5\sigma$  from a maximal  $\pi/4$ . The present value [12] is  $\tan^2\theta_{12} = 0.452_{-0.070}^{+0.088}$ , so the tribimaximal value is within the allowed range. With this identification eq. (2) becomes [10]

$$U_{\text{HPS}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -1/\sqrt{2} \\ -\sqrt{1/6} & \sqrt{1/3} & 1/\sqrt{2} \end{pmatrix} \quad (5)$$

	1 <sub>1</sub>	1 <sub>2</sub>	1 <sub>3</sub>	3
C <sub>1</sub>	1	1	1	3
C <sub>2</sub>	1	1	1	-1
C <sub>3</sub>	1	ω	ω <sup>2</sup>	0
C <sub>4</sub>	1	ω <sup>2</sup>	ω	0

**Table 1:** Character table of  $T \equiv A_4$ .  $\omega = \exp(2\pi i/3)$ .

	1 <sub>1</sub>	1 <sub>2</sub>	1 <sub>3</sub>	3
1 <sub>1</sub>	1 <sub>1</sub>	1 <sub>2</sub>	1 <sub>3</sub>	3
1 <sub>2</sub>	1 <sub>2</sub>	1 <sub>3</sub>	1 <sub>1</sub>	3
1 <sub>3</sub>	1 <sub>3</sub>	1 <sub>1</sub>	1 <sub>2</sub>	3
3	3	3	3	1 <sub>1</sub> + 1 <sub>2</sub> + 1 <sub>3</sub> + 3 + 3

**Table 2:** Kronecker products for irreducible representations of  $T \equiv A_4$

This ensures that the three mixing angles  $\theta_{ij}$  are consistent with present data, although more accurate experiments may require corrections to  $U_{\text{HPS}}$ .

The data allow a normal or inverted hierarchy, or a degenerate spectrum. The tribimaximal mixing,  $U_{\text{HPS}}$  of eq. (5), can accommodate all three of these neutrino mass patterns and so makes no prediction in that regard.

The success of  $U_{\text{HPS}}$  tribimaximal neutrino mixing has precipitated many studies of its group theoretic basis [8, 9, 11] and the tetrahedral group  $T \equiv A_4$  has emerged. The present analysis was prompted by earlier work of the present authors in systematically studying *all* non-abelian finite groups of order  $g \leq 31$  both for a quark flavor group [14] and for orbifold compactification in string theory [15]. Our question is whether or not  $T$  is singled out from these as the neutrino flavor symmetry?

The Kronecker products for irreducible representations for all the forty-five non-abelian finite groups with order  $g \leq 31$  are explicitly tabulated in the appendix of [15], where the presentation is adapted to a style aimed at model builders in theoretical physics rather than at mathematicians as in [13].

Study of [15] shows that a promising flavor group is  $\equiv SL_2(F_3)$ . The Kronecker products are identical to those of  $T \equiv A_4$  if the doublet representations are omitted and so the group  $SL_2(F_3)$  can reproduce successes of  $T$  model building. The use of  $SL_2(F_3)$  as a flavor group first appeared in [14] and then analysed in more details in [16].

$SL_2(F_3)$  has an advantage over  $T$  in extension to the quark sector because the doublets of  $SL_2(F_3)$ , absent in  $T$ , allow the implementation of a  $(2 + 1)$  structure to the three quark families, thus permitting the third heavy family to be treated differently as espoused in [18, 14, 17]

It is important to remark that  $T' \equiv SL_2(F_3)$  does not contain  $T \equiv A_4$  as a subgroup [13] so our discussion about quark masses does not merely extend  $T$ , but replaces it.

The philosophy used for  $SL_2(F_3)$  is reminiscent of much earlier work in [17, 18] where

	1 <sub>1</sub>	1 <sub>2</sub>	1 <sub>3</sub>	2 <sub>1</sub>	2 <sub>2</sub>	2 <sub>3</sub>	3
$C_1$	1	1	1	2	2	2	3
$C_2$	1	1	1	-2	-2	-2	3
$C_3$	1	$\omega^2$	$\omega^4$	-1	$\omega^5$	$\omega$	0
$C_4$	1	$\omega^4$	$\omega^2$	-1	$\omega$	$\omega^5$	0
$C_5$	1	1	1	0	0	0	-1
$C_6$	1	$\omega^2$	$\omega^4$	1	$\omega^2$	$\omega^4$	0
$C_7$	1	$\omega^4$	$\omega^2$	1	$\omega^4$	$\omega^2$	0

**Table 3:** Character table of  $T' \equiv SL_2(F_3)$ ,  $\omega = \exp(2\pi i/6)$ .

	1 <sub>1</sub>	1 <sub>2</sub>	1 <sub>3</sub>	2 <sub>1</sub>	2 <sub>2</sub>	2 <sub>3</sub>	3
1 <sub>1</sub>	1 <sub>1</sub>	1 <sub>2</sub>	1 <sub>3</sub>	2 <sub>1</sub>	2 <sub>2</sub>	2 <sub>3</sub>	3
1 <sub>2</sub>	1 <sub>2</sub>	1 <sub>3</sub>	1 <sub>1</sub>	2 <sub>2</sub>	2 <sub>3</sub>	2 <sub>1</sub>	3
1 <sub>3</sub>	1 <sub>3</sub>	1 <sub>1</sub>	1 <sub>2</sub>	2 <sub>3</sub>	2 <sub>1</sub>	2 <sub>2</sub>	3
2 <sub>1</sub>	2 <sub>1</sub>	2 <sub>2</sub>	2 <sub>3</sub>	1 <sub>1</sub> + 3	1 <sub>2</sub> + 3	1 <sub>3</sub> + 3	2 <sub>1</sub> + 2 <sub>2</sub> + 2 <sub>3</sub>
2 <sub>2</sub>	2 <sub>2</sub>	2 <sub>3</sub>	2 <sub>1</sub>	1 <sub>2</sub> + 3	1 <sub>3</sub> + 3	1 <sub>1</sub> + 3	2 <sub>1</sub> + 2 <sub>2</sub> + 2 <sub>3</sub>
2 <sub>3</sub>	2 <sub>3</sub>	2 <sub>1</sub>	2 <sub>2</sub>	1 <sub>3</sub> + 3	1 <sub>1</sub> + 3	1 <sub>2</sub> + 3	2 <sub>1</sub> + 2 <sub>2</sub> + 2 <sub>3</sub>
3	3	3	3	2 <sub>1</sub> + 2 <sub>2</sub> + 2 <sub>3</sub>	2 <sub>1</sub> + 2 <sub>2</sub> + 2 <sub>3</sub>	2 <sub>1</sub> + 2 <sub>2</sub> + 2 <sub>3</sub>	1 <sub>1</sub> + 1 <sub>2</sub> + 1 <sub>3</sub> + 3 + 3

**Table 4:** Kronecker products for irreducible representations of  $T' \equiv SL_2(F_3)$ .

the dicyclic group  $Q_6$  was used with doublets and singlets for the (1st, 2nd) and (3rd) families to transform as  $(\mathbf{2} + \mathbf{1})$  respectively. On the other hand,  $Q_6$  is not suited for tribimaximal neutrino mixing because like all dicyclic groups  $Q_{2n}$  it has no triplet representation. Recall that when the work on  $Q_6$  was done, experiments had not established neutrino mixing for the reason explained in our first paragraph.

To discuss the model building using  $SL_2(F_3)$  we must recall from the  $A_4$  model building [8, 9, 11] that the leptons can be assigned<sup>1</sup> to singlets and triplets as follows:

$$\left. \begin{array}{l} \left( \begin{array}{c} \nu_\tau \\ \tau^- \end{array} \right)_L \\ \left( \begin{array}{c} \nu_\mu \\ \mu^- \end{array} \right)_L \\ \left( \begin{array}{c} \nu_e \\ e^- \end{array} \right)_L \end{array} \right\} \begin{array}{ll} \tau_R^- & 1_1 \\ 3 \mu_R^- & 1_2 \\ e_R^- & 1_3 \end{array} \quad (6)$$

The symmetry breaking pattern of most interest is [13]

$$SL_2(F_3) \longrightarrow Q \longrightarrow Z_4 \longrightarrow Z_2 \longrightarrow \text{no symmetry} \quad (7)$$

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<sup>1</sup>An alternative assignment is in [19].

where  $Q$  is the quaternionic group so the first discussion concerns the vacuum alignment will cause the symmetry to break according to the pattern (7). Recall that the irreps of  $SL_2(F_3)$  are  $1_1, 1_2, 1_3, 2_1, 2_2, 2_3, 3$ .

By study of the character tables of these groups, we can ascertain the VEVs (Vacuum Expectation Values) which generate the required spontaneous symmetry breakdown.

The irreps of  $Q$  are  $1_1, 1_2, 1_3, 1_4, 2$ . Concerning the crucial first stage of symmetry breaking  $SL_2(F_3) \rightarrow Q$ , the irreps are related by

$$\begin{aligned}
 1_1 &\rightarrow 1_1 \\
 1_2 &\rightarrow 1_1 \\
 1_3 &\rightarrow 1_1 \\
 2_1 &\rightarrow 2 \\
 2_2 &\rightarrow 2 \\
 2_3 &\rightarrow 2 \\
 3 &\rightarrow 1_2 + 1_3 + 1_4
 \end{aligned} \tag{8}$$

so the breaking requires a VEV in  $1_2$  or  $1_3$  of  $SL_2(F_3)$ . We therefore assign the left-handed quarks, consistent with the  $2 + 1$  philosophy and the third family treated differently [17, 18] as follows:

$$\left. \begin{array}{l} \left( \begin{array}{c} t \\ b \end{array} \right)_L \\ \left( \begin{array}{c} c \\ s \end{array} \right)_L \\ \left( \begin{array}{c} u \\ d \end{array} \right)_L \end{array} \right\} \begin{array}{l} 1_1 \\ 2_1 \end{array} \tag{9}$$

and similarly the right-handed quarks are assigned as:

$$\left. \begin{array}{l} t_R \\ c_R \\ u_R \end{array} \right\} \begin{array}{l} 1_1 \\ 2_2 \end{array} \tag{10}$$

$$\left. \begin{array}{l} b_R \\ s_R \\ d_R \end{array} \right\} \begin{array}{l} 1_2 \\ 2_3 \end{array}$$

whereupon the mass matrices are:

$$U = \left( \begin{array}{c|c} \langle 1_3 + 3 \rangle & \langle 2_1 \rangle \\ \langle 2_3 \rangle & \langle 1_1 \rangle \end{array} \right) \tag{11}$$

and

$$D = \left( \begin{array}{c|c} \langle 1_2 + 3 \rangle & \langle 2_3 \rangle \\ \langle 2_2 \rangle & \langle 1_3 \rangle \end{array} \right) \tag{12}$$

To implement a hierarchy requires first a VEV to a  $SU(2)_L$ -doublet Higgs,  $H_{11}$  which is in the trivial singlet representation of  $SL_2(F_3)$ , thus giving a heavy mass to  $t$  without breaking  $SL_2(F_3)$ . This mass is naturally of order the weak scale  $\sim v/\sqrt{2} \sim 175$  GeV.

A VEV to a Higgs  $H_{12}$  breaks  $SL_2(F_3)$  to  $Q$  and can give masses to the  $b$  quark and  $c$  quarks. More explicitly, starting from the lagrangian for the  $SL_2(F_3)$  model, we have the Yukawa terms involving the top, bottom, and charm quarks:

$$Y_t \begin{pmatrix} t \\ b \end{pmatrix}_{11} t_{11} H_{11} \tag{13}$$

$$Y_b \begin{pmatrix} t \\ b \end{pmatrix}_{11} b_{12} H_{13} \tag{14}$$

$$\tag{15}$$

and

$$Y_c \left[ \begin{pmatrix} c \\ s \end{pmatrix}_{21} c_{22} + \begin{pmatrix} u \\ d \end{pmatrix}_{21} u_{22} \right] H_{13} + Y'_c \left[ \begin{pmatrix} c \\ s \end{pmatrix}_{21} c_{22} + \begin{pmatrix} u \\ d \end{pmatrix}_{21} u_{22} \right] H_3 \tag{16}$$

where the subscripts on the quark representations and Higgs doublets are the  $Sl_2(F_3)$  irreps where they live.

Hence, as stated above, the VEV for the  $H_{11}$  gives a mass to the top quark, but does not break  $SL_2(F_3)$ . The bottom and, in part, charm quark get their masses from a VEV for the Higgs  $H_{13}$  transforming according to the  $1_3$  irrep of  $SL_2(F_3)$ . Thus giving a VEV to  $H_{13}$  gives masses to these next heaviest quarks. This causes the family group to break from  $SL_2(F_3)$  to the quarternionic group  $Q$ . (As we shall show below all quarks can acquire a  $Q$  invariant mass). The  $b/c$  mass ratio is then simply

$$\frac{m_b}{m_c} = \frac{Y_b}{Y_c}. \tag{17}$$

The Yukawa couplings  $Y_{b,c}$  are free parameters and we can therefore get any  $m_b/m_c$  ratio we want. The theory is not predictive at this stage, but it is at least tunable. We can proceed this way to get the other quark mass ratios.

The remaining quark masses are generated from the following Yukawa terms

$$Y_s \left[ \begin{pmatrix} c \\ s \end{pmatrix}_{21} s_{23} + \begin{pmatrix} u \\ d \end{pmatrix}_{21} d_{23} \right] H_{12} + Y'_s \left[ \begin{pmatrix} c \\ s \end{pmatrix}_{21} s_{23} + \begin{pmatrix} u \\ d \end{pmatrix}_{21} d_{23} \right] H_3 \tag{18}$$

where  $s, d$  can get a mass from an  $H_{12}$  VEV and keep  $Q$  unbroken.

Hence, all the quarks can get masses from  $H_{12}$  and  $H_{13}$  VEVs that leave  $Q$  unbroken. Note that VEVs for  $H_3$ 's also contribute to masses of the first two quark generations, but not to  $t$  and  $b$  quark masses. More Higgses are needed to fill out all the off diagonal terms in the quark mass matrix. For instance, an  $H_{22}$  is needed to avoid a texture zero for  $m_{ut}$  if this is desired.

$$Y_{ut} \begin{pmatrix} t \\ b \end{pmatrix}_{11} u_{22} H_{23} \tag{19}$$

However, such a contribution is known phenomenologically to be very small.

However, it is important to observe that the only leptonic mass terms are from  $H_3$  VEVs, but under  $SL_2(F_3) \rightarrow Q$  we have

$$3 \rightarrow 1_2 + 1_3 + 1_4, \quad (20)$$

so a VEV for an  $H_3$  breaks  $Q$ , and giving multiple  $H_3$  VEVs can break  $Q$  to  $Z_4$ ,  $Z_2$ , or the trivial group of one element.

If we were to ignore  $\langle H_3 \rangle$  and off diagonal terms the hierarchy of quark masses becomes  $m_b/(m_c = m_u) = Y_b/Y_c$  and  $m_s = m_d$  and, because these relations are unsatisfactory,  $\langle H_3 \rangle$  must be significant which is interesting because, as mentioned above, it controls also the lepton masses. Although five quark masses remain encoded in parametric Yukawa couplings, the advantage over the minimal standard model is that the top quark mass is naturally large. As for all flavor groups, including the present one, the proliferation of Yukawa couplings  $Y_k$  is the principal obstacle to quantitative calculation of the quark masses.

To see that a reasonable lowest-order CKM matrix can be achieved rewrite  $m_i \equiv Y_k v_i$  where  $i$  is a  $T'$  representation and  $Y_k$  the appropriate Yukawa coupling. Then the quark mass matrices in eq. (11) and eq. (12) must be diagonalized. We work for simplicity in the limit

$$m_{2_1}^U = m_{2_3}^U = m_{2_2}^D = m_{2_3}^D = 0 \quad (21)$$

which corresponds to taking  $V_{ub} = V_{cb} = V_{td} = V_{ts} = 0$  in the CKM matrix. In this case all we need to do is diagonalize the  $(2 \times 2)$  sub-matrices,

$$\tilde{\mathcal{M}}_U = \left[ \begin{array}{c|c} im_3^U & m_{1_3}^U + \frac{m_3^U e^{-i\frac{\pi}{4}}}{\sqrt{2}} \\ \hline -m_{1_3}^U + \frac{m_3^U e^{-i\frac{\pi}{4}}}{\sqrt{2}} & m_3^U \end{array} \right], \quad (22)$$

$$\tilde{\mathcal{M}}_D = \left[ \begin{array}{c|c} im_3^D & m_{1_2}^D + \frac{m_3^D e^{-i\frac{\pi}{4}}}{\sqrt{2}} \\ \hline -m_{1_2}^D + \frac{m_3^D e^{-i\frac{\pi}{4}}}{\sqrt{2}} & m_3^D \end{array} \right]. \quad (23)$$

We begin by diagonalizing  $\tilde{\mathcal{M}}_U$ , which can be achieved by introducing two unitary matrices  $U_L$ , and  $U_R$  satisfying

$$U_L^\dagger \cdot \tilde{\mathcal{M}}_U \cdot U_R = \begin{bmatrix} m_u & 0 \\ 0 & m_c \end{bmatrix}, \quad (24)$$

or equivalently,

$$U_L^\dagger \cdot (\tilde{\mathcal{M}}_U \tilde{\mathcal{M}}_U^\dagger) \cdot U_L = \begin{bmatrix} m_u^2 & 0 \\ 0 & m_c^2 \end{bmatrix}, \quad (25)$$

where

$$\tilde{\mathcal{M}}_U \tilde{\mathcal{M}}_U^\dagger = \left[ \begin{array}{c|c} \frac{3}{2}[m_3^U]^2 + [m_{1_3}^U]^2 + m_{1_3}^U m_3^U & \sqrt{2} m_3^U m_{1_3}^U e^{-i\frac{\pi}{4}} \\ \hline \sqrt{2} m_3^U m_{1_3}^U e^{i\frac{\pi}{4}} & \frac{3}{2}[m_3^U]^2 + [m_{1_3}^U]^2 - m_{1_3}^U m_3^U \end{array} \right]. \quad (26)$$

From eqs. (26) and (25) the eigenvalues are calculated:

$$m_{u,c}^2 = \frac{(3[m_3^U]^2 + 2[m_{13}^U]^2) \mp 2\sqrt{3}m_3^U m_{13}^U}{2}. \tag{27}$$

which indicates how the quark mass spectrum can be successfully accommodated. The unitary matrix  $U_L$  takes the form:

$$U_L = \frac{1}{\sqrt{1+A^2}} \begin{bmatrix} 1 & e^{-i\frac{\pi}{4}}A \\ -e^{i\frac{\pi}{4}}A & 1 \end{bmatrix}, \tag{28}$$

with  $A = (\sqrt{3} + 1)/\sqrt{2}$ . We note that  $U_L$  is independent of quark masses.

In the down-sector, since the mass matrix  $\tilde{\mathcal{M}}_D$  takes the same form as  $\tilde{\mathcal{M}}_U$ , a matrix  $(\tilde{\mathcal{M}}_D \tilde{\mathcal{M}}_D^\dagger)$  is diagonalized by using the same unitary matrix  $U_L$  of eq. (28). Hence we reach the result for this special case

$$V_{\text{CKM}} = \begin{bmatrix} U_L^\dagger D_L | 0 \\ 0 | 1 \end{bmatrix} = \begin{bmatrix} U_L^\dagger U_L | 0 \\ 0 | 1 \end{bmatrix} = \mathbf{1}. \tag{29}$$

which is an acceptable first approximation to the CKM matrix.

For the neutrinos, as in earlier work on  $T$  [8], the masses are not uniquely predicted but the tribimaximal mixing angles [10] are. All these three neutrino mixing angles are consistent with existing measurements.

The Higgs VEVs with a commonality between quarks and leptons are in the  $H_3$  of  $T' \equiv SL_2(F_3)$  which has a simple decomposition under the quaternionic subgroup  $Q$  which is likely to play a key role in the goal of linking lepton masses with quark masses.

In summary, while  $T \equiv A_4$  is one candidate for a lepton flavor group which naturally gives rise to tribimaximal mixing, it is not unique among the non abelian finite groups in this regard. The choice  $T' \equiv SL_2(F_3)$ , also known as the binary tetrahedral group [20], satisfies the requirement equally well, and because it has doublet representations can thereby begin to accommodate the quark mass spectrum, particularly the anomalously heavy third family.<sup>2</sup> If our choice is the correct flavor symmetry, it remains to understand why Nature chooses the triplet representations for leptons and the doublet representations for quarks. Quantitative results for masses will require a relationship between the Yukawa parameters from our proposed symmetry.

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<sup>2</sup>Our  $SL_2(F_3)$  model appears to us as a promising framework worthy of further study with the goal to obtain quantitative results for fermion masses. When we first discussed  $SL_2(F_3)$  as a family symmetry in 1994 in [14], the state of neutrino physics at that time was insufficient to extend the model to the lepton sector. Subsequent data has now made this a realistic objective. Recent theoretical work [21] was a first step in this direction. A similar model has recently appeared [22] where tribimaximal neutrino mixing is included in an  $SL_2(F_3) \times Z_3$  family symmetry model with somewhat different assignments of quarks to  $SL_2(F_3)$  irreps, and different vacuum alignment. This model is also supersymmetric, so contains considerably more states and parameters. A grand unified theory based on  $T'$  is in [23].



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